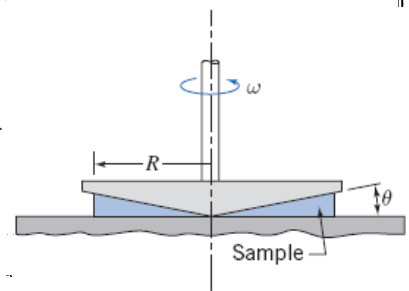


## Problem 2.67

[Difficulty: 4]

**2.67** The cone and plate viscometer shown is an instrument used frequently to characterize non-Newtonian fluids. It consists of a flat plate and a rotating cone with a very obtuse angle (typically  $\theta$  is less than 0.5 degrees). The apex of the cone just touches the plate surface and the liquid to be tested fills the narrow gap formed by the cone and plate. Derive an expression for the shear rate in the liquid that fills the gap in terms of the geometry of the system. Evaluate the torque on the driven cone in terms of the shear stress and geometry of the system.



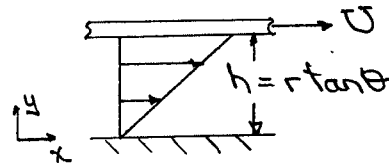
Solution:

Since the angle  $\theta$  is very small, the average gap width is also very small.

It is reasonable to assume a linear velocity profile across the gap and to neglect end effects.

The shear (deformation) rate is given by

$$\dot{\gamma} = \frac{du}{dy} = \frac{\Delta u}{\Delta y}$$



At any radius,  $r$ ,

the velocity  $U = \omega r$  and

the gap width  $h = r \tan \theta$

$$\therefore \dot{\gamma} = \frac{\omega r}{r \tan \theta} = \frac{\omega}{\tan \theta}$$

Since  $\theta$  is very small,  $\tan \theta \approx \theta$  and

$$\dot{\gamma} = \frac{\omega}{\theta}$$

Note: The shear rate is independent of  $r$ . The entire sample is subjected to the same shear rate.

The torque on the driven cone is given by

$$T = \int r dF \quad \text{where } dF = \tau_{yz} dA$$

Since  $\dot{\gamma}$  is a constant (for a given  $\omega$ ) then  $\tau_{yz} = \text{constant}$

and

$$T = \int r dF = \int_A r \tau_{yz} dA = \tau_{yz} \int_0^R r 2\pi r dr$$

$$T = \frac{2\pi}{3} R^3 \tau_{yz}$$